



PART II: Mathematical Methods for Engineering

Lecturers

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Dates: from Monday February 16th to Friday February 27th, 2015

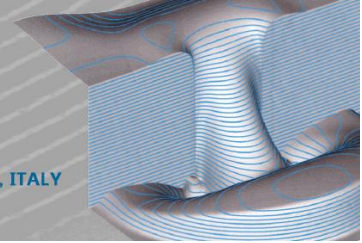
Venue: University of Trento, Department of Civil, Environmental and Mechanical Engineering (via Mesiano 77, I – 38123 Trento, Italy)

Summary

The second part of the winter school on numerical methods consists of a two-week intensive programme of 34 hours of theoretical lectures and 16 hours of computer laboratory exercises. The focus is on the presentation of some mathematical results that are at the basis of numerical approximation for partial differential equations, and on the analysis of some of these numerical procedures (boundary element, finite element and spectral methods). Theoretical results are described and proved (though not in complete detail), with the aim of placing the numerical methods on a solid ground and permitting their stability and convergence analysis. The algebraic structure of the discrete problems is also presented and analysed. The tutorials are devoted to the effective implementation in laboratory of these approximation schemes, using MATLAB and the software Freefem.

The course is mainly directed to PhD students and post-doctoral researchers in applied mathematics, engineering, and other scientific disciplines.





Contents

Theory

- Partial differential equations (elliptic equations, parabolic equations, hyperbolic equations, boundary value problems).
- Separation of variables (solution of heat and wave equations by means of Fourier expansion, orthonormal bases, Sturm-Liouville problems, Bessel functions, Legendre and Chebyshev polynomials).
- Fundamental solutions and Green functions for elliptic equations (Dirac delta "function", distributions, fundamental solutions, Green functions, integral representation formula in terms of the Green function).
- Integral equations and the boundary element method for elliptic problems (Green formulae, interior and boundary integral representation formulae in terms of the fundamental solution, integral equation on the boundary, collocation and Galerkin formulations of the boundary element method, algebraic structure of the approximating problems).
- Weak formulation and the finite element method for elliptic problems (minimization problems, Euler equation of a functional, weak formulation, Lax-Milgram lemma, existence and uniqueness of the solution, Galerkin approximation, finite element methods and spectral methods, family of triangulations and basis functions, Céa lemma and error estimates, mixed formulation and Stokes problem, mixed finite element methods, Ladyzhenskaya-Babuska-Brezzi condition and error estimates, compatible choices of finite elements, algebraic structure of the discrete problems, other applications).

Tutorials

- The boundary element method: remarks on programming.
- The finite element method, 1 (classical formulations) & programming
- The finite element method, 2 (mixed formulations) & programming
- FreeFEM: an example of finite element software.